MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

Arthur Crosfield Davies, Justice of the Peace for Co. Down, Lenaderg House, Banbridge, Co. Down, Ireland;
David Carder Davies, 19 Ashtead Road, London, E. 5;
Albert Downing C. Hooper, 42 High Street, Maldon, Essex;
Carl Emil Luplau Janssen, M.Sc., Director of Urania Observatory, Dr. Olgas Vej 25, Copenhagen F., Denmark; and
Lieut. William Henry Plummer, F.R.G.S., R.N.R., 19 Hawthorne Avenue, Swansea,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

A. J. R. Atkin, Department of Mines, P.O.B. 6231, Johannesburg, South Africa (proposed by R. T. A. Innes);
William Fraser, 212 Causewayside, Edinburgh (proposed by R. A. Sampson);
Lieut. Joseph Alfred Heenan, R.N.R., Dixie Lodge, Sandown Park, Wavertree, Liverpool (proposed by A. F. Butcher);
Edwin Henry Frank Lawrence, 2nd Officer, Mercantile Marine, Hollydene, 18 Waterloo Road, Southampton, Hants (proposed by W. G. W. Gilchrist); and
Rev. Daniel J. McHugh, C.M., Professor of Astronomy, De Paul University, 1010 Webster Avenue, Chicago, Illinois, U.S.A. (proposed by C. C. Conroy).

Seventy-five presents were announced as having been received since the last Meeting, including, amongst others :—

Aitken (John): Collected Scientific Papers (presented by the Royal Society of Edinburgh);

Lick Observatory: Three Transparencies of the Solar Corona, 1922 September 21 (presented by Professor Campbell);

Maginus (J. A.): Ephemerides, 1598–1630, 4 vols. (presented by Mr. L. J. Comrie); and

Rome, Specola Vaticana: A Preparatory Catalogue for a Durchmusterung of Nebulæ; first instalment, Zones 0° to 60° N.P.D. (presented by Rev. J. G. Hagen).

The Constitution of the Four Outer Planets.
By Harold Jeffreys, M.A., D.Sc.

It is widely believed that the four great planets are very hot, and that a large fraction of their volume is occupied by gas. In this paper it is suggested that the evidence hitherto advanced in support of this belief is inadequate, and that the data are more easily reconcilable with the hypothesis that these planets are cold and solid, their materials being of low density in comparison with terrestrial rocks.

In the first place, consider the amount of heat that any planet would have lost by radiation if its interior were in complete thermal communication with its surface and no heat had been received from outside. Take its radius to be a and its surface temperature V. Then the rate of loss of energy by radiation is $4\pi\sigma a^2 V^4$, where σ is Stefan's constant. If this continued for time T, and the mass of the planet was m, the total loss of heat per unit mass of the planet would be $4\pi\sigma a^2 V^4 T/m$. Considering Jupiter first, suppose that the surface temperature has been 300° absolute (27° C.) since it was formed. Its radius is 7×10^9 cm., and its mass 2×10^{33} grams. Stefan's constant is given by

$$\sigma(1^\circ)^4 = 5 \times 10^{-5} \text{ ergs/cm.}^2 \text{ sec.}$$

Jupiter is at least as old as the earth, and the earth is older than the oldest known rocks at its surface, whose age has been determined by the uranium/lead ratio method as $1{\cdot}6 \times 10^9$ years or 5×10^{16} seconds. Accordingly the amount of energy lost per gram of Jupiter's mass is found to be 6×10^{12} ergs, or $1{\cdot}5 \times 10^5$ calories. This would have been enough to solidify the planet on any reasonable hypothesis concerning its original temperature, and, if it solidified, the hypothesis that there is free thermal communication from the interior to the surface breaks down. Thus the surface would cool down till it was much cooler than the interior, and heat would be only slowly conducted out. The only datum in the above calculation that is capable of much variation is V. If the temperature was much lower than has been assumed, the loss of heat by radiation would be correspondingly reduced; but the suggestion

that Jupiter is cold at the surface is thereby admitted. The same is easily seen to hold from the other three great planets.

The supply of heat to the surface of the planets by solar radiation and by slow conduction from the interior must therefore be the controlling factors in determining the surface temperatures. Let us examine their relative importance. The amount of radiation received by the earth is about 7×10^{-3} cals./cm.^{2} sec., and the rate of conduction of heat from its interior is about 2×10^{-6} cals./cm.^{2} sec. If we temporarily suppose the external portions of the outer planets to be similarly constituted to those of the earth, cooling after solidification would take place in precisely the same way, would have proceeded to the same depth, and would be leading to the supply of just the same amount of heat to the surface. The amount of heat received from the sun is inversely proportional to the square of the distance, and therefore even at the distance of Neptune it would still exceed the rate of conduction from the interior of the planet. If we admit the possibility that the outer planets have very much more radio-active matter in their surface layers than the earth, it is possible that the heat received in this way may exceed that received by solar radiation; but it would have to be 10,000 times as great as on the earth to be comparable with the solar radiation received on the earth. Thus in any likely hypothesis the rate of receipt of heat per unit surface on any outer planet is much less than on the earth.

The surface temperature being determined by the condition that the rate of loss of heat is equal to the rate of receipt of it, would therefore be lower on the outer planets than on the earth, unless the temperature were considerably raised by selective absorption. For this to happen, these planets should absorb strongly in the visible region of the spectrum in comparison with the absorption for dark heat radiation. It is more natural to suppose that the effective radiating and absorbing layers of these planets are much colder than the corresponding layer on the earth.

The effective radiating and absorbing layer of an outer planet may reasonably be supposed to be, not the solid surface of the planet, if any, but a cloud layer at a height in the atmosphere corresponding to the height where cumulus and strato-cumulus clouds occur on the earth. Indeed, that what we see is not the surface of a solid body is clearly shown by the fact that it does not rotate as a rigid body, the period of rotation depending on the latitude. It has been claimed that this variation affords evidence that the atmospheres of the outer planets are very deep, but the evidence is not conclusive. The period of rotation of Jupiter, inferred from spots near the pole, is $9^h\ 55^m$, while the period for equatorial spots is $9^h\ 50^m$. Taking the radius of the planet as 70,000 km., we see that if the whole planet rotated in the period of polar spots, the additional velocity of equatorial spots necessary to give them their actual speed of rotation is 300 km./hr.; this is quite comparable with the velocity of an upper air wind on the earth, where velocities of 150 km./hr. over wide regions are frequent, and of 50 km./hr. normal. The winds thus inferred for Jupiter are of the nature of a general circulation, but unfortunately the theory of the general circulation of the atmosphere is very imperfect. We may, however, conjecture that the

quantities involved enter in the same way as in the case of the monsoons, the theory of which has been worked out for a homogeneous atmosphere.* It is found that if p' denotes the departure of the pressure from its mean value, g is the value of surface gravity, ρ' the departure of the density from its mean value ρ, and H the height of the atmosphere,

$$p' = \tfrac{1}{2} g \rho' \mathrm{H}.$$

If, further, v denotes the component of the wind velocity along a parallel of latitude, ω the angular velocity of the planet, a the radius, and θ the colatitude, we have for a steady motion the usual geostrophic condition

$$v = \frac{1}{2\omega\rho \cos\theta} \frac{\partial p'}{a\partial\theta}$$
$$= O\left(\frac{g\rho'\mathrm{H}}{4\omega\rho a}\right).$$

If suffixes E and J denote that the corresponding quantities refer to the earth and Jupiter respectively, we have roughly

$$\frac{v_\mathrm{J}}{v_\mathrm{E}} = 6, \quad \frac{g_\mathrm{J}}{g_\mathrm{E}} = 2, \quad \frac{\omega_\mathrm{J}}{\omega_\mathrm{E}} = 2{\cdot}5, \quad \frac{a_\mathrm{J}}{a_\mathrm{E}} = 11.$$

The ratio ρ'/ρ is probably nearly the same for Jupiter as for the earth ; for it is nearly equal to $-\mathrm{V}'/\mathrm{V}$, where V and V′ denote the mean absolute temperature and its anomaly, and in a planet whose surface temperature is determined wholly by radiation, the ratios of the temperatures at the different points are determined simply by the aspects these points present to the sun. Hence

$$\frac{\mathrm{H}_\mathrm{J}}{\mathrm{H}_\mathrm{E}} = 6\,\frac{a_\mathrm{J}}{a_\mathrm{E}},$$

so that there is no reason to infer that the depth of the homogeneous layer equivalent to the atmosphere of Jupiter is greater than $\frac{1}{150}$ of the radius. Actually, however, it is probable that the part of the atmosphere that rotates with the interior is an intermediate latitude, which requires this estimate to be reduced.

In the case of Saturn, observations of spots are much less numerous. Equatorial spots have rotated in $10^\mathrm{h}\ 14^\mathrm{m}$, and a spot in latitude 35° N. in $10^\mathrm{h}\ 38^\mathrm{m}$, the difference corresponding to an equatorial wind of 1300 km./hr. if the latter spot was at rest relatively to the interior. Now using the suffix S to denote that the corresponding quantity refers to Saturn, we have

$$\frac{v_\mathrm{S}}{v_\mathrm{E}} = 26, \quad \frac{g_\mathrm{S}}{g_\mathrm{E}} = 1{\cdot}1, \quad \frac{\omega_\mathrm{S}}{\omega_\mathrm{E}} = 2{\cdot}3, \quad \frac{a_\mathrm{S}}{a_\mathrm{E}} = 9.$$

Hence

$$\frac{\mathrm{H}_\mathrm{S}}{\mathrm{H}_\mathrm{E}} = 50\,\frac{a_\mathrm{S}}{a_\mathrm{E}},$$

so that the depth of the equivalent homogeneous atmosphere on Saturn may be $\frac{1}{20}$ of the radius.

* H. Jeffreys, *Phil. Mag.*, **34**, 449–458, 1917.

For Uranus and Neptune the required data are not available.

The densities of the outer planets are comparable with that of water. It has been held that their low densities afford evidence that large fractions of their volumes are gaseous, but the argument is open to suspicion. The mass of Jupiter is 21 times that of Uranus, yet their densities are nearly equal. It is very unlikely that, if much the larger portions of their volumes were in the gaseous state, the ratio of the volumes occupied by matter in the gaseous and solid or liquid states would in all cases be just great enough to make all four planets of roughly similar density. Thus the low density affords as good an argument for the hypothesis that these planets are solid as against it. It compels us, however, to suppose that these planets are all composed of matter very different from the chief constituents of the earth. On the tidal theory of the origin of the solar system this is plausible, for the inner planets would, on account of their low mass, have lost most of their lighter materials, whereas the outer planets would have retained them. The hypothesis that Jupiter, Uranus, and Neptune are mostly composed of ice agrees with their actual densities; but Saturn is less dense than ice even if we correct for the depth of its atmosphere suggested above. Some still lighter substance is therefore indicated.

The necessity of supposing that the outer planets are composed of something lighter than terrestrial rocks receives additional support when we consider their satellites. The cogency of the above argument about the depth of their atmospheres is imperfect in the absence of some better theory of the general circulation than has yet been put forward, but this difficulty does not affect the satellites, which are too light to retain any appreciable atmospheres. Taking the masses and diameters of the four inner satellites of Jupiter and of Titan to be as follows, the corresponding densities in terms of that of water have been found :—

Satellite.	Mass (Primary = 1).	Diameter (Miles).	Density.
JI.	1/22230	2470	2·7
JII.	1/39440	2060	2·6
JIII.	1/12500	3580	1·5
JIV.	1/22200	3360	1·0
Titan	1/4700	3500	1·4

The densities of the first and second satellites of Jupiter are comparable with those of terrestrial rocks, but those for the others are too small, and are again comparable with the density of ice. Data for the other satellites of the outer planets are not available; but Mr. Hepburn has found, in an unpublished paper which he has kindly shown me, that the inner satellites of Saturn also probably have low densities. The high albedo of Saturn's ring again suggests that it is composed of some colourless substance in a state of fine division, possibly ice or some other light non-metallic compound.

If the solid parts of Jupiter and Saturn have densities similar to those of their denser satellites, about half of their volumes would have to be gaseous, and thus the depths of their atmospheres would have to be about one-fifth of their radii. There seems to be no fatal objection

to this suggestion, in view of the uncertainty in the theory of the general circulation. The evidence does, however, indicate that such atmospheres must be cold and surround solid surfaces, and it suggests that the suspended clouds in them are probably composed of some material with much lower melting and boiling points than water. The above argument relating to the temperature refers to the effective radiating surface, which is the cloud surface, and it may be suggested that this may be cold without the solid surface being so, the temperature within the atmosphere decreasing downwards as in the case of the earth. The vertical variation of temperature in the earth's atmosphere, however, arises from the fact that the earth is heated by the sun's radiation absorbed by the solid surface. In the great planets the cloud layers must be practically opaque, so that the solid surfaces cannot be heated directly, and therefore there is reason to believe that the atmospheres below the cloud layers are approximately isothermal.

Note on the Constitution of the Spiral Nebulæ. By F. A. Lindemann.

Recent measurements by van Maanen of the rate of rotation of spiral nebulæ tend to show that these are not, as is often supposed, extra-galactic island universes. It may therefore be worth while to formulate briefly some arguments in favour of an alternative theory set down some years ago, but not hitherto published, as the occasional appearance in spiral nebulæ of so-called novæ seemed difficult to reconcile with this hypothesis.

The spiral nebulæ have in general continuous spectra intersected by dark lines. They must, therefore, be composed either of stars, stones, or dust. The significant facts concerning them, apart from their form, the newly measured rotation and the above-mentioned "novæ" are, their high receding velocities (of the order of $5 \cdot 10^7$ to 10^8 cm./sec.), their avoidance of the plane of the galaxy and the important fact discovered by Reynolds and Seares, that the centre is much redder than the edges.

The first alternative, that they are composed of stars and form island universes is in accord with their spectra, is not in disagreement with their forms, and allows of a plausible explanation of the appearance in them from time to time of bright objects of about 16th to 18th magnitude, which are interpreted as novæ analogous to those appearing in the galaxy. The other facts enumerated above are, however, difficult to reconcile with this view.

Firstly, there is no reason why the island universes should in general recede from the galaxy, and even if this could be explained gravity could scarcely account for the observed high velocities. Since the mean angular diameter of the largest spirals is scarcely of the order of 30′, the density of matter in space on the island-universe theory would be barely 10^{-6} of that in the galaxy. As the potential energy and consequently the square of the velocity is proportional to the product of the mean density and the square of the radius, a universe 10^4 times the

mean distance between neighbouring nebulæ would have to be postulated containing some 10^{12} island universes, in order to account for speeds ten times greater than galactic speeds.

Secondly, there is no reason why the island universes should avoid the plane of the galaxy, a comparatively insignificant object compared to the whole system. The suggestion that those in the plane of the galaxy are obscured by hypothetical dark nebular masses surrounding the galaxy is, of course, a complete though scarcely plausible reply.

The most difficult point is to explain on the island-universe theory why the centres of spirals are redder than the peripheries. The exact reverse is what might have been expected. In our galaxy, and presumably in others, a rough sort of equipartition of energy appears to hold, so that bright hot blue stars of large mass move slower than small cold red stars. Hence in the oscillations and orbits of the components of the system one would expect the red dwarfs with their high velocities in general to attain the greatest amplitudes, whilst the slow massive blue stars should seldom depart far from the centre of gravity of the system. One might, therefore, expect a blue nucleus and red edges, whereas Reynolds has shown that in reality the exact opposite obtains.

The second hypothesis, that the spirals consist of stones, has all the above difficulties without any countervailing advantages, but the assumption that they consist of dust has much in its favour.

Particles whose diameter is of the order 10^{-4} cm. are repelled by the sun in consequence of radiation pressure with a force considerably greater than its gravitational attraction. The same holds good, of course, of other stars, so that there must be a continuous stream of fine dust leaving the neighbourhood of the stars and moving towards regions of low radiation density. Though never truly stable within the galaxy, the particles would tend to remain in these regions for a longer time than elsewhere, and would probably form clouds on account of their mutual gravitational attraction. The mutual attraction of two particles of masses m_1 and m_2 and radii R_1 and R_2 at a distance r would be $\frac{m_1 m_2 f}{r^2}$, and would, of course, only outweigh their repulsion if the radiation density were low. If u is the radiation density the repulsion is $\frac{2\pi R_1^2 R_2^2 u}{4r^2}$, so that radiation pressure balances gravitational attraction for two equal particles of radius R_1 and density ρ_1, when $R_1^2\rho_1^2 = \frac{9u}{32\pi f}$. In dealing with small particles and densities this has to be multiplied by a function of the reflection coefficient k, for though all the incident energy is re-emitted in the steady state, the wave-length of the temperature radiation is large compared to the linear dimensions of the particles, so that only the reflected or scattered light is effective in repelling the particles from one another. In the neighbourhood of the sun u, apart from the sun's own radiation, is about $4{\cdot}7 \cdot 10^{-15}$. In dark regions one may therefore probably put $u = 10^{-15}$. If k is

about 60 per cent., therefore $R_1\rho_1$ is about $2 \cdot 10^{-5}$, *i.e.* particles of density 2 will attract one another even if their radii are 10^{-5} cm. Hence in these dark regions the dust particles will tend to form into clouds, which will contract until their relative kinetic energy equals half their potential energy.

Under repulsive forces from the stars, of course such a cloud can never be really stable. Sooner or later it will escape from its dark space and be expelled from the galaxy. On its course it will pass through regions of higher radiation density, and the light scattered will increase, and the mutual repulsion will approach and may even outweigh the gravitational attraction of the smaller particles. Under the influence of their initial rotation and the mutual repulsion, and aided probably by tidal forces, the cloud may assume the form of a spiral.

It is evident on this theory that the motion of spiral nebulæ should be predominantly receding. It is easy to show that very high velocities might be attained. The light pressure upon a particle of appropriate size may rise to eight times the gravitational attraction in the case of the sun. For stars radiating more per unit mass it may be considerably greater. But even if one took the solar type as the mean, velocities of recession eight times the maximum gravitational galactic velocities could be accounted for, and this is the order of velocities observed.

Again, it is evident that these clouds of dust in escaping from the galaxy would tend to avoid the plane of the Milky Way. Without accurate knowledge of the form of the galaxy it is useless to enter into any detailed calculation, but it is certain that the proportion of spirals in low latitudes would be small.

It is clear that this hypothesis is bound up with the acceptance of the reflection theory for luminosity of these nebulæ.* There does not seem any very adequate objection to this. The luminosity of a nebula per square degree would be $\frac{\pi}{360^2} kn\mathrm{L}$ where n is the equivalent number of stars of zero magnitude representing all the star-light, and L the luminosity of one of them. In the neighbourhood of the sun $n = 350$, so that the stellar magnitude of a white nebula would be $2 \cdot 5 \log \frac{360^2}{\pi n}$, or $5 \cdot 2$ magnitude per square degree. It is clear that this would be increased proportionally if the nebula were more strongly illuminated, but it is not quite certain that much more intense intrinsic luminosities occur.

The reflection theory, of course, explains the observed spectra. Further, on the above dust-cloud hypothesis it explains the distribution of red and blue light in the nebulæ. As shown above, the first particles to be ejected and those most strongly repelled by the nucleus will be those for which R_1 is smallest, *i.e.* the smallest particles. By hypothesis all the particles are of a diameter of the order of the wave-length of light, otherwise they would have been swept up by the stars. Therefore

* Mr. R. d'E. Atkinson has called my attention to the fact that the observed velocities will be twice as great as the true velocities if the nebular light is reflected galactic light.

the central particles will on the whole be larger than the average wave-length, the peripheral particles smaller. Now light of a certain wave-length can only be materially reflected by particles which are of greater linear dimensions than this wave-length. Therefore the peripheral particles will reflect the blue end of the spectrum mainly, whilst the nucleus will reflect the whole spectrum. In other words, the centre will be redder than the edges.

The difficulty confronting this view of spirals has hitherto been the appearance, from time to time, in them of temporary bright objects of the 16th to 18th magnitude, which on the island-universe theory are assumed to be novæ. It is possible to account for these on the dust-cloud hypothesis, though admittedly the explanation is somewhat artificial.

Suppose the nebula receding from the galaxy at a high velocity encounters a comet-like cloud of stones or sand. The collisions with the nebular dust particles will heat and finally evaporate the stones, and the interspace between them will rapidly fill up with gas. This gas will maintain its high velocity relative to the nebula until collisions with the nebular particles bring it to rest, and a large amount of energy will be generated and radiated. Seen from a distance the whole phenomenon will appear as a bright object which appears suddenly and gradually dies away, *i.e.* it will exhibit the typical light-curve of a nova.

In order to show that this hypothesis leads to consistent results when treated quantitatively, we will examine the mechanism in detail, inserting rough values of observable quantities where necessary, so as to check the order of magnitude of our results. If the mean density of the nebula is ρ_0, the heat generated per second on each stone of radius R_2 will be $\pi R_2^2 \frac{\rho_0 v^3}{2}$. If l_2 is the latent heat per gramme of these stones and ρ_2 their densities, the whole mass will be vaporised in a time $t_2 = \frac{8 R_2 \rho_2 l_2}{\rho_0 v_0^3}$. In meteorites l_2 is of the order $2 \cdot 10^{10}$ and ρ_2 about 2. If we use these values and put $v_0 = 10^8$, the mean density of the nebula $\rho_0 = 3 \cdot 10^{-13} \frac{R_2}{t_2}$, or if we put $t_2 = 3 \cdot 10^6$, $\rho_0 = 10^{-19} R_2$. According to our hypothesis the nebula is distant from us about the diameter of the galaxy, say, 10^{22} cm. In order to appear as a star of the 18th magnitude at a distance of 10^{22} cm., about eleven thousand light years, the cloud must produce and radiate an energy $E = 2 \cdot 6 \cdot 10^{33}$ ergs per second. Hence if S_2 is the cross-section of the vaporised cloud of stones, $\rho_2 \cdot \frac{\rho_0 v^3}{2} = 2 \cdot 6 \cdot 10^{33}$, whence $S_2 = \frac{5 \cdot 2 \cdot 10^{28}}{R_2}$.

The retardation of this mass of gas will be given by $\frac{dv}{dt} = \frac{S_2 \rho_0 v^2}{2 M_2}$, M_2 being its mass, or $\frac{1}{v} - \frac{1}{v_0} = \frac{S_2 \rho_0}{2 M_2} t$. Hence the velocity will be reduced to one-half in a time $t' = \frac{2 M_2}{S_2 \rho_0 v_0}$, v_0 being the initial velocity. When the

velocity has sunk to one-half, the radiation per second will, of course, be reduced to one-eighth, *i.e.* the bright object will have diminished in lustre by 2·26 magnitudes. If this time can be determined empirically the mass of the intruding comet-like cloud is given by $M_2 = \frac{Et'}{v_0^2}$. Putting $t' = 10^7$ seconds, which is presumably of the right order of magnitude, this makes $M_2 = 2{\cdot}6 \cdot 10^{24}$, *i.e.* about one-thirtieth of the moon's mass.

From S_2 and M_2 it is obviously easy to determine the mean distance d between the stones in the cloud in terms of R_2 and ρ_2. Assuming the clouds and the stones to be spherical

$$d_2 = \left(\frac{S\,12\pi}{81}\right)^{\frac{1}{3}} \frac{S_2^{\frac{1}{3}}\rho_2^{\frac{1}{3}}R_2}{M_2^{\frac{1}{3}}} \text{ or } d_2 = 3{\cdot}4 \cdot 10^6 R_2^{\frac{1}{3}},$$

using the values assumed above. It is evident that the interstices between these stones would be filled up rapidly once evaporation became appreciable. On the other hand, apart from extraneous disturbances, such clouds would be relatively stable objects since the mean free path of each stone would be about $\frac{16}{9\sqrt{2\pi}} \cdot \frac{S^{\frac{2}{3}}}{M_2} R_2\rho_2 = \frac{1{\cdot}6 \cdot 10^{18}}{R_2^{\frac{1}{3}}}$, *i.e.* collisions would be extremely rare. Incidentally, the assumption we have made throughout implicitly, that S_2 can be treated as constant, is clearly justified, since the cloud's radius is of the order 10^{14}, so that the gas expanding for 10^7 seconds would not materially enlarge its dimensions.

The density of the cloud of gas formed in this way by the evaporation of the stones would be $\rho_2' = \frac{3\sqrt{\pi M_2}}{4S^{\frac{3}{2}}} = 2{\cdot}9 \cdot 10^{-19} R_2^{\frac{3}{2}}$. It is necessary to show that this density is sufficient materially to decelerate the particles of the nebula in a time of the order of the time in which the "novæ" are known to disappear. If R_1 is the radius and ρ_1 the density of the particles we have $\frac{dv}{dt} = \frac{3}{8} \cdot \frac{\rho_2' v^2}{R_1\rho_1}$ or $\frac{1}{v} - \frac{1}{v_0} = \frac{3}{8} \frac{\rho_2'}{R_1\rho_1} t$. The time t' within which v has been reduced to $v_0/2$, so that the luminosity has diminished to one-eighth, must be the same as the time t' derived above. After this time clearly

$$8R_1\rho_1 = 3\rho_2' v_0 t' = 3 \cdot 2{\cdot}9 \cdot 10^{-19} R_2^{\frac{3}{2}} v_0 t'.$$

Inserting the values assumed above, *i.e.* $t' = 10^7$ and $v_0 = 10^8$ and putting $R_1 = 5{\cdot}10^{-5}$, as is implicit in our whole theory, with $\rho_1 = 2$ therefore $R_2 = 0{\cdot}95$. In other words, the cloud of stones postulated to account for the objects in spiral nebulæ, generally called novæ, must consist of fragments of about 1 cm. radius; incidentally this is probably about the size of the particles forming the head of a comet. If the possibility is conceded that such clouds can exist, the so-called novæ may be little more than analogues to terrestrial meteoric showers.

It remains to consider the implications for the nebula of the density ρ_0 which was found above. If $R_2 = 1$ cm., $\rho_0 = 10^{-19}$ and the mass per

square centimetre over a distance of 10^{18} cm. about one light year, would be 0·1 gramme. This seems to be of the right order for the opacity of spiral nebulæ. The total mass would be large. If one takes a typical spiral to be 200″ in diameter and 20″ thick, and its distance to be 10^{22} cm., its mass would be less than 10^{37}. It might well be considerably less than this, as the mean density is probably a good deal less than the density in the brightest parts where "novæ" usually occur, but it seems that its mass must be at least of the same order as that of the sun, and may well be considerably greater. The number of particles in unit volume, n_1 is, of course, given by $\frac{3\rho_0}{4\pi R_1^3 \rho_1}$, so that if $R_1 = 5 \cdot 10^{-5}$ and $\rho_1 = 2$, $n_1 = 9{\cdot}6 \cdot 10^{-8}$ and the mean free path is $\frac{1}{\sqrt{2} \cdot 4\pi R_1^2 n} = 2{\cdot}4 \cdot 10^{14}$. Since the mean relative velocity of the particles in the spiral is probably low, therefore the number of collisions is comparatively small, and a spiral nebula might well be a relatively stable object unless the particles are quite inelastic.

It is true that the above explanation seems somewhat artificial, but apparently there is no insuperable objection to it. In view of the difficulties of the island-universe theory, therefore, it appeared worth while to attempt to formulate the alternative hypothesis as a basis for discussion.

The above may be summarised as follows:—

The island-universe theory of spiral nebulæ accounts only for their spectra, form, and the appearance of the "novæ."

The dust-cloud theory accounts for the spectra, form, velocities, positions, and for the distribution of colour in spiral nebulæ. Collision with comet-like bodies must be invoked to account for the so-called "novæ."

The Greenwich Observations of the Moon (1751–1922.) By Sir Frank Dyson and Dr. A. C. D. Crommelin. (Plate 13.)

Dr. P. H. Cowell contributed a series of papers on these observations to *Monthly Notices*, **64, 65, 66** (1903, 1904, 1905, 1906), in which he applied various theoretical corrections to the tabular places of the Moon, and analysed the resulting values of "tabular – observed," both to search for new terms and to correct the coefficients of the known ones. Since the supersession of Hansen's Tables by those of Brown marks the close of a chapter in the study of the Moon's motion, it has been decided to publish in collected form the errors of the tabular longitudes and latitudes of the Moon from the beginning of 1851 when the Transit Circle was erected, to the end of 1922. Cowell's analysis extended to 1901 June 18, this being the end of No. 133 of his "periods" of 400 lunar days. The interval from 1901 June 18 to 1922 December 31 contains exactly 19 additional "periods"; these have been corrected by the

application of the same terms to the tabular places as those used by Cowell for the earlier periods.

The terms applied to the Hansen-Newcomb longitudes were :—

$$
\begin{aligned}
&+ 1''\cdot562 \sin D \\
&- \cdot893 \sin (g + 2\varpi - 2J) \\
&- \cdot676 \sin (g + 2\varpi - 3V - 5E) \\
&- \cdot548 \sin g \\
&- \cdot246 \sin (2D - g) \\
&+ \cdot55 \sin g' \\
&- \cdot55 \sin (g - D) \\
&+ \cdot316 \sin (g + 2\varpi - 3J + 7^\circ) \\
&+ (N + \Delta g + 4''\cdot06 \sin \Omega)\, 2e \cos g.
\end{aligned}
$$

g, g' being the mean anomalies of Moon and Sun, ϖ the mean longitude of the lunar perigee, D the mean elongation of the Moon from the Sun, V, E, and J the mean longitudes of Venus, Earth, and Jupiter. The numerical expressions for the above terms are given in *Monthly Notices*, **64**, 421.

N denotes Newcomb's correction to Hansen's mean longitude of the Moon. It is to be taken as zero from 1883 onwards, since Newcomb's corrections are then included in the *Nautical Almanac*. $\Delta g = 4''\cdot63 - 13''\cdot99T + 4''\cdot743T^2$, where T is reckoned in centuries from 1800·0, $2e = 0\cdot1098$.

Also the observations for the whole interval from 1851 to 1922 have been corrected as follows :—

(1) The right ascensions have been reduced to the system of Newcomb's Fundamental Catalogue by applying the systematic corrections given in that catalogue, in addition to the corrections applied to the individual Greenwich *annual* catalogues to reduce them to the Greenwich General Catalogues.

(2) The declinations have been corrected by the quantities given by Newcomb in *American Ephemeris Papers*, **2**, 450–2. For the observations after 1887 tables have been made by comparing the successive Greenwich Catalogues with that of Newcomb.

(3) The obliquity of the ecliptic has been reduced throughout to Newcomb's value by the table in *American Ephemeris Papers*, **9**, 47.

(4) Hansen's parallax has been increased by 0″·40 (*American Ephemeris Papers*, **9**, 44). This correction is practically the same as that used by Brown.

(5) For the period 1862 to 1883, both inclusive, a correction has been applied to the longitude residuals of $-\frac{1}{30}$ (original tab. − obs. long.) to correct for the wrong tabular mean time used in forming the ephemeris at transit. This correction had already been applied before 1862, and from 1884 onwards.

1. The following is a specimen of the manner in which it is proposed to print the resulting residuals, which are in units of 0″·1 :—

Date.	Lunar Day.	Longitude Tab. – Obs.	E.N.P.D. Tab. – Obs.
1851 Jan. 8	35,401	– 21	+ 9
9	402	– 10	+ 11
18	411	– 47	– 22
21	414	– 49	– 9
22	415	+ 11	– 7
24	417	– 27	– 20
26	419	– 27	+ 50
31 18h ●			
Feb. 8	431	– 6	– 4
10	433	– 54	– 59
11	434	– 50	0
13	436	– 14	+ 12

The times of New Moon, denoted by the black disc, are inserted to the nearest hour. It is thus easy to group by lunations, and the "lunar day" is an aid in analysis. This aid is fully realised when the observations are arranged with an interval of one line to the lunar day, lines being left blank when no observations were made. As an example, the combined corrections (1), (2), (3), (4), were calculated for each calendar year as functions of the Moon's longitude, using the position of the node at the middle of the year. When the observations are spaced in the manner described, the mean longitude is indicated by position alone, and the corrections can be very rapidly inserted.

2. In discussing the residuals a preliminary step has been the reduction from the Hansen-Newcomb values of tabular longitude to those of Professor Brown, utilising the tables given by the latter in *M.N.*, **73**, 692–714, and in *Astron. Journal*, 799. The quantity called δT by Brown has also been applied, since it produces a notable improvement in the residuals of the last 15 years, while its effect before this is small.

It arises from the adoption of a secular acceleration 4″·79 in excess of Brown's value, which includes solely the part arising from the change in the eccentricity of the Earth's orbit. There are consequential changes in the mean motion and the great empirical term, which were investigated by Dr. Fotheringham (*M.N.*, **80**, 304 *et seq.*).

The expression for δT given by Brown is :—

$$\delta T = -2''\cdot55 + 2''\cdot02T + 4''\cdot79T^2 + 13''\cdot60 \sin(139^\circ T + 104^\circ\cdot2) - 10''\cdot71 \sin(140^\circ T + 100^\circ\cdot7),$$

T being measured in centuries from 1800.

Our work indicated that the value $-2''\cdot71$ for the constant term gives better results than the value $-2''\cdot55$; the former has therefore been adopted in the tables that follow.

Since the residuals tab. – obs. for the first 89 of Cowell's periods of 400 lunar days are given by Brown (*M.N.*, **73**, 703–705), under

the heading "Mer" it suffices to give for these periods "δT" and "Mer+δT," the latter being the adopted value of tab.−obs. long. The unit is ″·01. Brown gives two values for periods 86, 87, 88, 89. The mean of the two has been used, as being likely to introduce less discontinuity between the preceding and following values.

Tabular – Observed Longitudes for Periods 1 *to* 89. (*Unit* ″·01.)

Period.	δT.	"Mer"+δT.	Period.	δT.	"Mer"+δT.	Period.	δT.	"Mer"+δT.
1	−23	+227	31	+ 7	−123	61	−35	+365
2	−20	− 80	32	+ 6	−114	62	−37	+293
3	−18	− 68	33	+ 6	− 84	63	−39	+231
4	−17	− 27	34	+ 5	−295	64	−41	+199
5	−15	−105	35	+ 5	−245	65	−43	+117
6	−13	− 13	36	+ 4	−276	66	−45	+ 75
7	−11	+ 49	37	+ 3	−237	67	−47	+ 93
8	−10	+160	38	+ 2	−268	68	−49	+ 81
9	− 8	+122	39	+ 1	−259	69	−51	+139
10	− 6	+174	40	0	−140	70	−53	+ 17
11	− 5	+ 65	41	− 1	+139	71	−55	+ 95
12	− 3	+177	42	− 3	+ 37	72	−57	+163
13	− 2	−112	43	− 4	− 74	73	−58	+222
14	− 1	+ 49	44	− 5	+ 25	74	−60	+160
15	0	+ 60	45	− 6	+ 4	75	−62	+158
16	+ 1	+ 31	46	− 8	+ 72	76	−63	+167
17	+ 2	− 98	47	− 9	+ 61	77	−64	+166
18	+ 3	−187	48	−11	+149	78	−66	+174
19	+ 4	− 16	49	−12	+188	79	−67	+ 83
20	+ 4	+ 84	50	−14	+ 76	80	−68	+122
21	+ 5	− 15	51	−16	+ 94	81	−69	+ 51
22	+ 6	− 94	52	−18	+ 22	82	−70	− 10
23	+ 6	−144	53	−20	+ 30	83	−71	− 41
24	+ 7	+ 7	54	−21	+199	84	−72	−132
25	+ 7	− 33	55	−23	+207	85	−73	− 43
26	+ 7	−273	56	−25	+215	86	−73	−113
27	+ 7	−113	57	−27	+373	87	−74	−224
28	+ 7	+ 7	58	−29	+201	88	−74	−189
29	+ 7	−193	59	−31	+249	89	−74	−199
30	+ 7	−203	60	−33	+227			

The following table gives (*a*) Cowell's residuals, or the residuals following Cowell's method, after the application of corrections (1) to (5) given on p. 360. (*b*) Brown's correction to reduce from the Hansen-Newcomb system to his own; this is taken from *M.N.*, **73**, 705–706,

and from *Astron. Journal*, 799, for Cowell's periods 142 to 152. (*c*) The equinox correction given in *M.N.*, **73**, 700. Since we had already applied this correction in (1) on p. 360, we have to take it out from (*b*) by applying it with the sign given on p. 700; since this is the sign for application to observed place, and our quantities are tab. − obs. (*d*) The quantity δT with $-2''\cdot71$ substituted for $-2''\cdot55$. (*e*) The adopted value of tab. − obs. longitude, being the sum of the four preceding columns; the unit is $''\cdot01$.

Tabular – Observed Longitudes for Periods 90 *to* 152. (*Unit* $''\cdot01$.)

Period.	*a.*	*b.*	*c.*	*d.*	*e.*	Period.	*a.*	*b.*	*c.*	*d.*	*e.*
90	−245	− 40	+36	−74	−323	122	+ 33	+189	+44	+ 37	+303
91	−292	+ 24	+36	−74	−306	123	− 28	+210	+56	+ 44	+282
92	−329	+ 23	+36	−74	−344	124	+ 9	+160	+56	+ 53	+278
93	−287	− 44	+33	−73	−371	125	+ 97	+153	+56	+ 61	+367
94	−352	+ 28	+12	−72	−384	126	+ 28	+256	+65	+ 69	+418
95	−462	+150	+12	−71	−371	127	− 59	+331	+74	+ 78	+424
96	−499	+195	+12	−70	−362	128	− 109	+347	+74	+ 87	+399
97	−550	+233	+12	−69	−374	129	− 166	+413	+74	+ 97	+418
98	−645	+316	+12	−68	−385	130	− 247	+466	+74	+108	+401
99	−612	+244	+33	−66	−401	131	− 242	+390	+75	+117	+340
100	−533	+121	+36	−64	−440	132	− 211	+307	+81	+128	+305
101	−427	+ 50	+36	−62	−403	133	− 257	+307	+81	+139	+270
102	−357	+ 44	+36	−60	−337	134	− 301	+282	+81	+150	+212
103	−238	− 18	+36	−58	−278	135	− 316	+269	+28	+162	+143
104	−165	− 41	+36	−55	−225	136	− 395	+311	0	+174	+ 90
105	−214	+ 46	+36	−52	−184	137	− 522	+355	0	+186	+ 19
106	−208	+ 61	+41	−49	−155	138	− 549	+309	0	+198	− 42
107	− 66	− 3	+51	−46	− 64	139	− 545	+274	0	+211	− 60
108	− 62	+ 15	+51	−43	− 39	140	− 633	+323	0	+224	− 86
109	− 33	+ 87	+51	−39	+ 66	141	− 710	+363	0	+238	−109
110	+ 44	+102	+51	−34	+163	142	− 818	+372	0	+251	−195
111	+ 42	+149	+51	−29	+213	143	−1088	+468	0	+266	−354
112	− 63	+198	+51	−25	+161	144	−1138	+573	0	+281	−284
113	−190	+375	+50	−20	+215	145	−1254	+588	0	+296	−370
114	−172	+333	+44	−15	+190	146	−1267	+572	0	+311	−384
115	−168	+323	+44	− 9	+190	147	−1365	+597	0	+326	−442
116	−185	+322	+44	− 3	+178	148	−1366	+580	0	+342	−444
117	− 83	+231	+44	+ 3	+195	149	−1275	+527	0	+358	−390
118	− 10	+171	+44	+ 8	+213	150	−1305	+509	0	+375	−421
119	− 32	+225	+44	+15	+252	151	−1288	+486	0	+392	−410
120	+ 24	+240	+44	+23	+331	152	−1311	+462	0	+408	−441
121	+ 21	+185	+44	+30	+280						

The final columns of this and the preceding table are plotted in Plate 13 (after the application of the corrections given on p. 367).

3. In subjecting the tab.−obs. longitudes to analysis, the first step taken was to draw the periodogram for various assumed periods. Up to periods of about 40 years the analysis was by single 400-lunar-day periods; for the last 19 lines of the table they were combined in groups of five, periods 151, 152 being excluded. In the following table C, S denote the coefficients of the cosine and the sine respectively. In a few cases near the beginning of the table a few additional values, supplied by extrapolation after period 152, were inserted to facilitate analysis. In all cases the zero angle is taken at the middle of the period analysed, that is, about the year 1835.

Periodogram Analysis.

Period in Years.	C.	S.	$(C^2+S^2)^{\frac{1}{2}}$.	Period in Years.	C.	S.	$(C^2+S^2)^{\frac{1}{2}}$.
	″	″	″		″	″	″
7·93	−·160	−·038	·164	45·34	+·293	+·984	1·027
11·34	−·135	+·030	·138	42·50	− ·30	+1·07	1·111
12·47	−·118	+·115	·165	44·35	+ ·02	+1·03	1·030
13·60	+·018	−·222	·223	46·37	+ ·39	+0·77	·863
15·87	−·112	−·160	·195	48·59	+ ·71	+ ·49	·863
17·00	−·059	−·051	·078	51·00	+ ·97	− ·03	·970
18·13	+·274	−·430	·510	53·69	+1·17	− ·75	1·390
18·6	+·524	−·035	·525	56·67	+1·29	−1·37	1·882
22·67	−·097	−·190	·213	60·00	+1·29	−1·83	2·239
24·94	−·197	−·126	·234	63·76	+1·07	−2·21	2·455
27·20	−·119	−·629	·640	68·00	+ ·73	−2·51	2·614
28·33	−·007	−·682	·682	72·87	+ ·38	−2·63	2·657
29·47	−·412	−·487	·638	78·47	+ ·09	−2·21	2·212
31·74	+·184	−·837	·857	85·00	− ·140	−1·826	1·832
34·00	+·432	−·145	·456	92·72	− ·307	−1·256	1·293
29·15	+·224	+·646	·684	102·00	− ·353	− ·875	·944
31·39	+·633	+·271	·689	113·32	− ·275	− ·475	·549
34·00	+·440	−·136	·461	127·5	− ·166	− ·186	·249
37·09	−·130	+·142	·193	136·0	− ·084	− ·068	·108
40·80	−·301	+·910	·958	145·7	− ·021	+ ·021	·030

The periodogram is shown as a continuous curve in fig. 2.

If only one harmonic is used to express the residual, the periodogram indicates the maximum ordinate at 71·60 years. The corresponding formula is:—

$$+0''\cdot42\cos\theta - 2''\cdot63\sin\theta \text{ where } \theta \text{ vanishes at the date } 1835\cdot71 \quad \text{(i)}$$

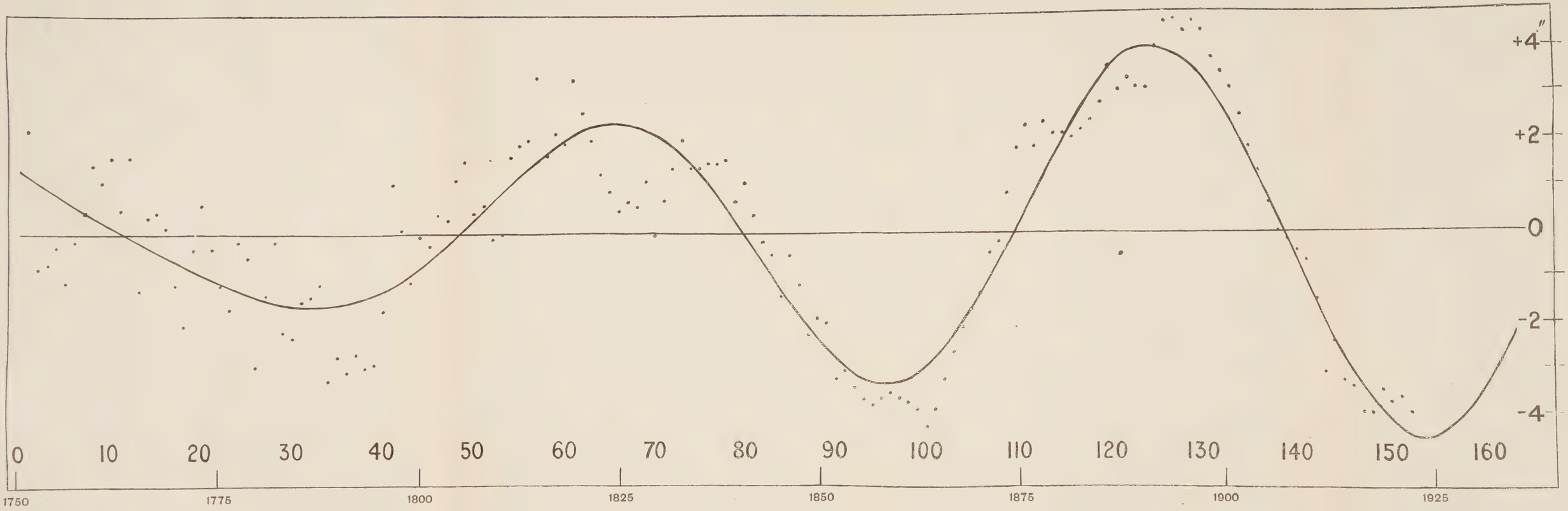

Fig. 1.—Residuals (Tabular—Observed) of the Moon from 1750 to 1922. The Tabular places are those of Brown, with the addition of δT (Corrected).

The residuals are given in the table on p. 366 ; the sum of their squares is $37''\cdot94$.

4. It is clear from the increase in the size of the oscillations shown in Plate 13 that a much better solution can be obtained by using two harmonics. The shape of the periodogram, with the fairly flat summit of the largest peak extending from 60^y to 72^y, also suggests that it arises from two terms. The residuals, after taking out the above harmonic, show maxima near 1776, 1838, 1895, and minima near 1793, 1861, 1918, indicating a period of about 61 years.

However, since two terms of such closely adjacent periods are not separated in the analysis, it was necessary to carry out a least-square solution with the two terms combined. This was done for several successive trial periods, until in a final solution the corrections to the periods were themselves introduced as unknowns. It was then found that the solution was somewhat indeterminate. The amplitude of the

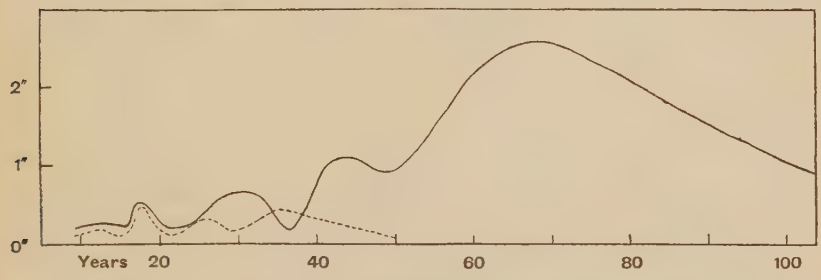

Fig. 2.—Periodogram —— Before, - - - After } taking out long-period terms.

largest term was therefore assumed from a previous solution, and the remaining unknowns determined from the normal equations.

The solution is

$$\left.\begin{aligned} &-0''\cdot45\cos\theta - 2''\cdot76\sin\theta \\ &+1''\cdot59\cos\phi + 0''\cdot28\sin\phi \end{aligned}\right\} \quad . \quad . \quad . \quad \text{(ii)}$$

when θ, ϕ are both zero at 1835·71, and increase respectively $5^\circ\cdot82$ and $6^\circ\cdot90$ in 400 lunar days; the periods in years are 70·10 and 59·13. To illustrate the range of solutions that give nearly as good residuals, formula (iii) is inserted as an alternative to (ii) :—

$$\left.\begin{aligned} &-0''\cdot32\cos\theta - 3''\cdot03\sin\theta \\ &+1''\cdot51\cos\phi + 0''\cdot61\sin\phi \end{aligned}\right\} \quad . \quad . \quad . \quad \text{(iii)}$$

when θ, ϕ are zero at 1835·71, and increase $5^\circ\cdot92$ and $6^\circ\cdot96$ respectively in 400 lunar days.

The following table gives the comparison between tab. − obs. and the formula (i), (ii), (iii). The unit is $0''\cdot01$.

Comparison with Formulæ (i), (ii), (iii).

Periods.	(T. – O.)—Formula. T. – O.	(i).	(ii).	(iii).	Periods.	(T. – O.)—Formula. T. – O.	(i).	(ii).	(iii).
1– 5	– 11	– 247	– 94	– 124	76–80	+ 142	+ 166	+ 95	+ 89
6–10	+ 98	– 50	+ 56	+ 39	81–85	– 35	+ 121	+ 68	+ 46
11–15	+ 48	+ 23	+ 51	+ 47	86–90	– 210	+ 25	+ 29	+ 23
16–20	– 37	+ 67	+ 12	+ 20	91–95	– 355	– 89	– 37	– 37
21–25	– 56	+ 153	+ 37	+ 51	96–100	– 392	– 160	– 81	– 77
26–30	– 155	+ 110	– 26	– 13	101–105	– 285	– 142	– 71	– 64
31–35	– 172	+ 80	– 23	– 18	106–110	– 6	+ 12	+ 40	+ 47
36–40	– 236	– 55	– 96	– 91	111–115	+ 194	+ 83	+ 46	+ 51
41–45	+ 26	+ 44	+ 120	+ 121	116–120	+ 234	+ 21	– 75	– 70
46–50	+ 109	+ 44	+ 125	+ 126	121–125	+ 302	+ 36	– 87	– 90
51–55	+ 110	– 70	+ 29	+ 31	126–130	+ 412	+ 162	+ 53	+ 49
56–60	+ 253	+ 65	+ 79	+ 83	131–135	+ 254	+ 78	+ 33	+ 29
61–65	+ 241	– 21	+ 8	+ 12	136–140	– 16	– 75	– 31	– 29
66–70	+ 81	– 130	– 152	– 168	141–145	– 262	– 190	– 58	– 47
71–75	+ 160	+ 54	– 9	– 11	146–150	– 416	– 231	– 47	– 33

The residuals of solution (ii) taken in groups of 6 give the following means: +″·06, +″·39, +″·06, −″·30, −″·23. These can be improved by an alteration in Fotheringham's value of the great empirical term; it was further noticed that Brown used Newcomb's mass of Venus, which both Cowell and Ross have shown to be too small (from discussion of observations of the Sun and Mars respectively). A correction of + 1/80th of Brown's value of the great Venus term was applied, and a least-square solution gave the following formula to be used instead of Fotheringham's: $+13''\cdot28 \sin(138^\circ\cdot3T + 104^\circ\cdot11)$, the period being 260·3 years instead of 259·0.

The constant ″·09 is added, in order to keep the mean residual zero; this cancels part of the −″·16 applied earlier.

The formula now obtained in place of (ii) is

$$\left.\begin{aligned} &-0''\cdot37\cos\theta - 3''\cdot07\sin\theta \\ &+1''\cdot55\cos\phi + 0''\cdot58\sin\phi \end{aligned}\right\} \quad . \quad . \quad . \quad \text{(iv)}$$

θ, ϕ are zero at the date 1835·71, and increase respectively 5°·86 and 6°·92 in 400 lunar days. Their periods are 69·63 years, and 58·96 years; one gains a revolution on the other in 385 years. They are in unison in 1961·3, in opposition 1768·8.

The following table shows the corrections applied for the changes in the Venus term and the great empirical term, and the residuals (T. – O.) minus (iv), unit ″·01 :—

Comparison with Formula (iv).

Periods.	Correction.	Residual.	Periods.	Correction.	Residual.
1– 5	+ 1	−114	76–80	−13	+76
6–10	− 6	+ 45	81–85	− 8	+55
11–15	−11	+ 49	86–90	− 2	+25
16–20	−16	+ 14	91–95	+ 4	−29
21–25	−21	+ 37	96–100	+10	−60
26–30	−25	− 34	101–105	+16	−41
31–35	−28	− 43	106–110	+21	+75
36–40	−30	−127	111–115	+27	+82
41–45	−31	+ 83	116–120	+32	−44
46–50	−31	+ 86	121–125	+36	−63
51–55	−30	− 7	126–130	+39	+72
56–60	−29	+ 47	131–135	+42	+51
61–65	−26	− 20	136–140	+44	− 5
66–70	−22	−178	141–145	+45	−19
71–75	−18	− 31	146–150	+45	+ 8

The means of groups of 6 are now $''\cdot00$, $+''\cdot06$, $-''\cdot12$, $-''\cdot03$, $+''\cdot07$, showing a notable improvement. The sum of the squares of the residuals is $12''\cdot98$, about one-third of that obtained with a single harmonic term. The solution (iv) is the curve drawn in the diagram (Plate 13).

5. A new periodogram was now drawn for the residuals of solution (iv), with the following result:—

Period.	Amplitude.	Period.	Amplitude.	Period.	Amplitude.	Period.	Amplitude.
^{y} 28·33	$''\cdot250$	^{y} 31·87	$''\cdot452$	^{y} 36·42	$''\cdot508$	^{y} 42·50	$''\cdot358$
29·14	·246	32·90	·503	37·78	·484	44·34	·313
30·00	·299	34·00	·537	39·23	·460	46·36	·251
30·91	·378	35·17	·534	40·79	·416	48·58	·186

This periodogram is shown by the dotted curve in fig. 2.

It will be seen that the maximum at 43^{y} in the original periodogram has disappeared, showing that it does not represent a real term, but arises from the larger terms. The above table shows a maximum at about the half-period of the 69·63-year term. Also the maximum at 18·6 years in the original periodogram is not much affected by the removal of the terms in (iv). A least-square solution was made with assumed periods 34·815 years and 18·62 years (the period of revolution of the Moon's nodes).

The following terms were obtained:—

$$\left.\begin{aligned} &+''\cdot32\cos\theta+''\cdot40\sin\theta \\ &+''\cdot27\cos\phi+''\cdot31\sin\phi \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (v)$$

θ, ϕ are zero at the date 1835·71, and increase 11°·72, 21°·92 respectively in 400 lunar days. The following are the residuals after applying (v). They refer to the same periods as solution (iv) above. Unit ″·01.

Residual.	Residual.	Residual.
+23	+ 4	− 8
+73	+ 52	+ 1
−10	+ 2	+ 6
− 7	−101	− 14
+29	− 28	− 27
−51	− 13	+ 77
+21	+ 30	+ 83
−50	+ 55	− 27
+44	− 25	−108
+29	− 25	+ 6

The sum of the squares of the residuals is now reduced to 6″·0, and the terms (v) produce an improvement for the greater part of the range of observations; but for the last 25 years they appear to make things worse instead of better. Our confidence in these terms was further diminished by forming the periodograms of residuals separately for the two half-periods: viz. for the years 1750–1835, and 1835–1920. That for 1750–1835 shows maxima for periods of 18·6 years and 39·2 years (amplitudes ″·66 and ″·81 respectively); that for 1835–1920 shows a maximum at 27 years (amplitude ″·56); there is a slight kink in the curve at 18·6 years (amplitude ″·32), but no maximum. Further, a periodogram derived from the observations from 1851–1922, the period in which Hansen's Tables were used, shows no irregularity near 18·6 years. It seems, therefore, that the series reduced by Airy, extending from 1750 to 1850, does require a further correction to the figure-of-Earth term (18·62 years period), but this correction is not required in the following series. We therefore prefer to adopt (iv) as the definitive solution; with regard to (iv) it is well to remark that the difference between the lengths of the two periods is fairly well determined, but that the periods themselves may each be altered by some 2 years, with corresponding adjustment of coefficients, without greatly increasing the sum of the squares of the residuals.

6. It is of interest to compare our formula with others that have been arrived at. For convenience of comparison the angular motion of the argument is given in degrees per annum, and t is the interval in years from the zero epoch 1900·0. Also our sign is reversed to make the expression a correction to tabular longitude.

Our formula (iv) becomes $3''\cdot09\sin(5^\circ\cdot170t+339^\circ\cdot17)+1''\cdot66\sin(6^\circ\cdot106t+281^\circ\cdot83)$.

Cowell's formula (*M.N.*, 65, 34) is $2''\cdot35\sin(5^\circ\cdot426t+323^\circ\cdot47)+''\cdot73\sin(8^\circ\cdot602t+79^\circ\cdot95)$.

Radau's formula (Tables, Introd., p. 92) is $3''\cdot3\sin(5^\circ\cdot6t+333^\circ\cdot6)$.

Ross's formula (*M.N.*, 72, 27) is $2''\cdot9\sin(6^\circ\cdot316t+350^\circ\cdot6)+''\cdot8\sin(15^\circ\cdot65t+313^\circ)$.

Dale's formula (*M.N.*, 74, 645) is $2''\cdot96\sin(6^\circ\cdot41t+355^\circ\cdot4)+1''\cdot11\sin(18^\circ\cdot8t+73^\circ\cdot4)$.

Tustin's formula (*Astron. Journ.*, 734) is $7''\cdot30\sin(5^\circ\cdot7t+8^\circ\cdot15)+4''\cdot03\sin(7^\circ\cdot1t+233^\circ\cdot45)+''\cdot50\sin(17^\circ\cdot1t+15^\circ\cdot45)$.

The last three formulæ are all deduced from Newcomb's discussion of occultations (*Ast. Papers Amer. Eph.*, 9, pt. i.), and assume his value of the great empirical term and secular acceleration. Tustin's formula is deduced from only 63 years' observations (1853–1916). Brown (*M.N.*, 73, 694) says, "it seems clear that there must be at least three terms with periods between 75 and 35 years, and coefficients between 3″·0 and 0″·5." This is, however, qualified by the sentence, "if the residuals can be represented in such a manner at all." Fotheringham, in a letter dated 1919 December 14, wrote: "There is a minor fluctuation with period 60 to 70 years; discovered by Cowell, confirmed by others. The amplitude of the term was 1″ from 1662 to 1795, now it is nearly 5″. Perhaps it is a combination of two long-period terms with nearly equal period." The description agrees fairly well with formula (iv).

Fotheringham calls attention to a paper by Ross, in *Astron. Journ.*, 29, 152–6, which gives diagrams of the errors of the Sun for the same period as that dealt with for the Moon. His curve is for obs. – tab., and so has to be reversed to compare with ours. When that is done there is a general agreement in the positions of maxima and minima, though not in their relative magnitudes; the correspondence is not sufficiently close to carry conviction that a variation in the Earth's rotation is the cause of these terms, but close enough to suggest its possibility.

7. We give predicted values for the correction required by Brown's Tables at 2·5-year intervals up to 1950, as accuracy of prediction is the best test of an empirical formula; but warned by similar attempts we are prepared for failure. δT is based on Brown's formula. "Corrn." is the joint correction arising from change in the Venus term, change in the great empirical term, and a constant −″·07; *a*, *b* are the corrections arising from the terms of periods 69·63 and 58·96 years respectively. The sum is the total correction to Brown's mean longitude.

Date.	δT.	Corrn.	*a*.	*b*.	Sum.
	″	″	″	″	″
1922·5	+4·26	+·28	+3·08	+1·43	+9·05
25·0	+4·65	+·28	+2·93	+1·60	+9·46
27·5	+5·05	+·27	+2·64	+1·66	+9·62
30·0	+5·48	+·26	+2·21	+1·61	+9·56
32·5	+5·91	+·25	+1·67	+1·44	+9·27
35·0	+6·37	+·24	+1·05	+1·17	+8·83
37·5	+6·84	+·22	+·37	+·81	+8·24
40·0	+7·32	+·21	−·32	+·40	+7·61
42·5	+7·81	+·19	−1·00	−·04	+6·96
45·0	+8·32	+·17	−1·63	−·48	+6·38
47·5	+8·85	+·15	−2·18	−·88	+5·94
50·0	+9·38	+·13	−2·61	−1·22	+5·68

The observed correction to Brown's Tables at the date 1922·5 is +8″·47.

It may be mentioned that the periodogram shows no trace of the presence of any term having the period of the sunspot cycle; this was examined because some of the values found for the period of the 70-year term are close to 6 sunspot cycles; but a connection between the two could not be inferred in the absence of any trace of influence of the single cycle.

Examination was also made of a period of 428 to 430 days, but no term was found.

Royal Observatory, Greenwich:
1923 *April* 12.

The Secular Acceleration of the Moon's Mean Motion as determined from Occultations and Conjunctions in the "Almagest" (a Correction).
By J. K. Fotheringham, M.A., D.Litt.

In *M.N.*, **75** (1915), 377–94, Miss Longbottom and I published an investigation into the secular acceleration of the Moon's mean motion, based on occultations and conjunctions recorded in the *Almagest*. Herr Schoch of Heidelberg, who has recently constructed MS. tables for the rapid computation of the places of the Sun, Moon, and planets, and for other purposes useful to students of history and ancient literature, and who has included in his collection tables for determining the Moon's parallax in latitude and longitude, has informed me that he tested his tables on the seventh observation in our series and found that the correct parallax in longitude should be $+0^{\circ}\cdot37$, instead of $+0^{\circ}\cdot141$, which we had found. He then checked with his tables all our parallaxes in latitude and longitude, but found no other error. The error is important, and I am obliged to Herr Schoch for drawing my attention to it.

I have traced the error to its source, and have deduced the following corrections to figures and statements published on pp. 384–92 of our paper:—

P. 384, line 19, column VII., *for* $215^{\circ}\ 40'\ 33''$ *read* $215^{\circ}\ 54'\ 8''$.
" 20, " *for* $+1^{\circ}\ 20'\ 21''$ *read* $+1^{\circ}\ 25'\ 14''$.
" 24, " *for* $+296''$ *read* $+300''$.
" 25, " *for* $-11''$ *read* $-21''$.
" 26, " *for* $+0''\cdot12$ *read* $-0''\cdot11$.
P. 385, " 3, " *for* $215^{\circ}\ 40'\ 28''$ *read* $215^{\circ}\ 54'\ 3''$.
" 4, " *for* $+1^{\circ}\ 5'\ 11''$ *read* $+1^{\circ}\ 10'\ 4''$.

P. 390, lines 4–8, *for* "I find . . . 74"" *substitute* "I find that the line joining π Scorpii and δ Scorpii, as affected by refraction, would intersect the parallel of latitude of the south cusp, also affected by refraction, in longitude $215^{\circ}\ 47'\ 51''$, making an angle of $83^{\circ}\ 54'\ 18''$ with the parallel. Refraction would reduce the apparent longitude of π Scorpii, δ Scorpii, and the south cusp by $49''$, $44''$, and $41''$ respectively.'

P. 390, line 8, *for* $+519''$ *read* $-328''$.

P. 391, last line, *for* $+\frac{519}{270} = +1''\cdot92$ *read* $-\frac{328}{270} = -1''\cdot21$."

" *for* 296 *read* 300.

" *for* $\pm 0''\cdot464$ *read* $\pm 0''\cdot470$.

P. 392, line 9, *for* $\pm 0''\cdot464$ *read* $\pm 0''\cdot470$.

" 10, *for* $+4''\cdot9$ *read* $+4''\cdot7$.

" 13, *for* $\pm 1''\cdot00$ *read* $\pm 1''\cdot09$.

The observations have been weighted for errors in time-keeping, not only on each of the three theories given on pp. 392, 393 of our paper, but also on each of three theories suggested in my paper, "The Probable Error of a Water-Clock," *Classical Review*, 29 (1915), 236–8. The six theories discussed in that paper are:—

(I.) That errors of an equal amount are equally probably at all hours of the night.

(II.) That the time shown by the clock was corrected so as to give to each hour its average length for the particular night.

(III.) That the constant hourly error of the clock is negligible, but that there is a variable error, and that the observed time is always compared with sunset.

(IV.) That the constant hourly error is negligible, but that there is a variable error, and that the evening observations are compared with sunset and the morning observations with sunrise.

(V.) That the variable error is negligible, but that there is an error accumulating in proportion to the interval in time from sunset.

(VI.) That the variable error is negligible, but that there is an error proportionate to the interval separating evening observations from sunset and morning observations from sunrise.

The correction to the assumed value of $+8''\cdot0$ for secular acceleration on these six theories now works out as follows:—

I.	$+2''\cdot2$	$\pm 0''\cdot94$	IV.	$+2''\cdot0$	$\pm 0''\cdot86$
II.	$+2\cdot2$	$\pm 0\cdot93$	V.	$+1\cdot8$	$\pm 0\cdot78$
III.	$+1\cdot9$	$\pm 0\cdot82$	VI.	$+1\cdot7$	$\pm 0\cdot86$

The close agreement of the results obtained on these widely different theories shows that the conclusion obtained is virtually independent of the theory by which the weights are distributed among the different observations.

The correction to the seventh observation makes it difficult, however, to retain that preference for the sixth theory which I expressed in my two papers. It will be found that with the corrected figures three out of the four morning observations show greater errors in the observed time in proportion to their distance from sunrise than any of the three evening observations show in proportion to their distance from sunset. This renders it probable that the simple precaution of reading the clock-time of sunrise was generally or always omitted, and that the probable error in an observed time was in proportion to the interval of time since

the clock was set going at sunset. It will be observed that the fifth theory, which makes this assumption, is now the one which gives the smallest probable error in the value of the acceleration.

But it must be admitted that two of the three evening observations show greater errors in the observed time in proportion to their distance from sunset than any of the four morning observations. This is what one would expect, because the effect of the variable error, which I have not attempted to determine, diminishes in proportion to the constant hourly error as the night proceeds. I have decided to select the fifth theory as being perfectly simple, and as giving on the whole the least discordant results.

The following table ought, in consequence, to take the place of that given at the head of p. 393 :—

No. of Obs.	Factor.	Weight.	No. of Obs.	Factor.	Weight.
I.	$156\cdot54 \times \frac{385}{398}$	4	V.	$39\cdot29 \times \frac{322}{262}$	43
II.	$33\cdot51 \times \frac{367}{406}$	109	VI.	$197\cdot09 \times \frac{348}{298}$	2
III.	$47\cdot70 \times \frac{355}{439}$	67	VII.	$198\cdot12 \times \frac{300}{270}$	2
IV.	$156\cdot58 \times \frac{459}{468}$	4			

From the total probable error of $\pm 0''\cdot78$ found with these weights, I allow $\pm 0''\cdot13$ for the probable error apart from the clock errors, and find for the probable error due to the clock errors $\pm 0''\cdot77$. Gathering up the errors from all causes as in the previous paper, I get revised weights which give a correction to secular acceleration of $+2''\cdot0 \pm 0''\cdot74$. This gives $\pm 0''\cdot73$ for the probable error due to the clock errors. With this revised value I have re-computed the probable error of each observation and its appropriate weight, and have formed the following table, which should be substituted for the second table on p. 393 :—

No. of Observation.	Probable Error from Causes (1) and (2).	Probable Error from Cause (3).	Total Probable Error.	Weight.
I	$\pm 0''\cdot195$	$\pm 5''\cdot340$	$\pm 5''\cdot344$	2
II.	$0''\cdot324$	$1''\cdot068$	$1''\cdot116$	53
III.	$0''\cdot353$	$1''\cdot360$	$1''\cdot405$	34
IV.	$0''\cdot374$	$5''\cdot416$	$5''\cdot429$	2
V.	$1''\cdot152$	$1''\cdot703$	$2''\cdot056$	16
VI.	$0''\cdot660$	$8''\cdot118$	$8''\cdot145$	1
VII.	$0''\cdot470$	$7''\cdot763$	$7''\cdot778$	1

With these weights the correction to the assumed secular acceleration becomes $+2''\cdot1 \pm 0''\cdot74$. As the probable error is the same as that

found in the previous approximation, there is no reason for amending the weights further. The probable clock error corresponding to the probable error in each observation attributed to clock error is 7·6 minutes an hour, a far less startling result than the probable error of 12 minutes an hour which I found in my previous paper.

Adding the correction to the assumed acceleration, I get for the total sidereal acceleration of the Moon's mean motion, $+10''\cdot1 \pm 0''\cdot74$. This requires a further correction of $+0''\cdot2$ to counterbalance the correction of $+5''$ by which the centennial mean motion of the Moon, which I found in *M.N.*, 81 (1920), 125, for epoch 1800 January 0·0, exceeds the centennial motion assumed in our paper on the occultations in the *Almagest*. It would appear, therefore, that the occultations and conjunctions recorded in the *Almagest* are best satisfied by a sidereal secular acceleration of the Moon amounting to

$$+10''\cdot3 \pm 0''\cdot74 \text{ per century.}$$

University Observatory, Oxford
1923 *April* 12.

Occultations of Stars by the Moon, observed at the University Observatory, Oxford, 1923 January 27.

(*Communicated by the Director.*)

Phenomenon.	Oxf. Sid. T.	G.M.T.	Observer.
	h m s	h m s	
θ' Tauri, dis.	4 46 23·1	8 27 2·4	F.A.B.
Aldebaran, dis.	8 54 49·3	12 34 47·9	H.H.T.
,, reap.	9 49 47·7	13 29 37·3	H.H.T.

Notes.

θ' Tauri, sharp disappearance, but lunar halo.

Aldebaran, reappearance sudden, but star seemed distinctly within the moon's edge ; small and red, growing quickly.

Initials: H.H.T., Professor Turner; F.A.B., F. A. Bellamy.

Latitude $+51° 45' 34''$. Longitude $0^h 5^m 0^s\cdot4$ W.

Telescope : 12·2-in. visual. The performance of the O.G. has gradually deteriorated, and after these observations it was dismounted and taken to Messrs. Cooke, Troughton & Simms, at York, for examination and repair.

Comet Pons-Winnecke: Perturbations by Jupiter. By C. J. Merfield.

An investigation of the movements of this comet was commenced recently, and will deal with the returns during the period 1915 to 1927.

The perturbations by Jupiter have been determined for the 1921 to 1927 revolution, and are available.

As these results, so far obtained, may be of interest to Fellows, this part of the investigation is here given.

In the determination of these perturbations, the method of variation of parameters has been adopted. Only perturbations due to the first powers of the disturbing force have been taken into account. Owing to the conditions of the problem, the second or higher powers will have little effect during the two revolutions being investigated.

The provisional orbit elements used in the calculation are as follows:—

Epoch of Osculation, 1921 *June* 29.

$M = 2^\circ\ 38'\ 6''\cdot53.$

T = 1921 June 12·92205 G.M.T.

$\left.\begin{aligned} \pi &= 268^\circ\ 24'\ 1'' \\ \Omega &= 98\ 7\ 24 \\ \iota &= 18\ 55\ 35 \end{aligned}\right\}$ 1921.

$\phi = 43\ 15\ 18$

$\log e = 9\cdot8358469$

$q = 0\cdot0173904$

$a = 0\cdot5194200$

$\mu = 590''\cdot0334$

$\rho = 2196\cdot53$ days.

The preceding data have been prepared from the two sets of orbit elements by Mr. Frank E. Seagrave, and published in the *A.J.*, No. 812.

Perturbations by Jupiter, Limits 1921 *June* 29 *and* 1927 *July* 16.

$\left.\begin{aligned} \delta M &= -\ 15'\ 52''\cdot44 \\ \delta\pi &= +\ 2\ 36\ \cdot41 \\ \delta\Omega &= -\ 2\ 49\ \cdot03 \\ \delta\iota &= +\ 0\ 40\ \cdot30 \\ \delta\phi &= +\ 1\ \ 0\ \cdot78 \\ \delta\mu &= +\ 0''\cdot5249 \end{aligned}\right\}$ Equinox 1921.

Applying these perturbations to the provisional elements, and reducing, to the equinox 1927, those that determine the position of the orbit, we obtain the following system of osculating orbit elements:—

Epoch of Osculation, 1927 *July* 16.

$M = 4^\circ\ 15'\ 27''\cdot84.$

$T = 1927$ June $20^d\cdot04517$ G.M.T.

$\left.\begin{aligned} \pi &= 268^\circ\ 31'\ 39''\cdot4 \\ \Omega &= 98\ \ 9\ 29\ \cdot1 \\ \iota &= 18\ \ 56\ 14\ \cdot1 \end{aligned}\right\}$ 1927.

$\phi = 43\ \ 16\ 18\ \cdot78.$

$\log e = 9\cdot8359828$

$q = 0\cdot0168369$

$a = 0\cdot5191625$

$\mu = 590''\cdot5583$

$\rho = 2194\cdot53$ days.

It may be noted that the perturbations here presented are very similar in magnitude to those deduced by K. Hillebrand, *A.N.*, 4787, for the period 1909 December 3 to 1915 September 3, with the exception of the variation of "M," the mean anomaly.

The above orbit elements indicate that this comet will pass through the descending node on June 28. Two days later the Earth will pass the comet's line of nodes. The near approach of the comet to the Earth about this time should occasion considerable interest; the circumstances will be much better than those of 1921. Dr. Crommelin has referred to this subject in the *Journal* of the B.A.A., 1922 November.

The proposed investigation will be continued during the time available for such work. When completed it will be possible to determine the elements of the undisturbed orbit with greater accuracy.

Appendix.

The present occasion presents an opportunity of referring to the very simple formula

$$z = \int_{-\omega}^{+\omega} y\,dx = \frac{\omega}{3}(y_0 + 4y_1 + y_2),$$

which may be expanded thus

$$z = \frac{\omega}{3}(y_0 + 4y_1 + 2y_2 \ \ldots\ 4y_{n-1} + y_n),$$

"ω" being the interval of the argument, "n" to be an even number, equal to or greater than the number two.

The value of this simple formula seems to have been overlooked in many text-books dealing with mechanical integration.

It has a very wide application in many astronomical problems. Indeed, the approximate numerical value of a large number of integrals may be found by its use. The formula will give accurate results when the curve is one of the third or lower degree.

In any tabulation of a function where the third differences are constant or nearly so, then this simple formula can be depended upon to give very accurate results; in any case, the result will be quite as good as any found by the use of finite differences.

If the functions "y" are taken to represent areas then the formula will give the cubical content. Under these conditions it is much used by engineers and surveyors, but they rarely use the formula for the computation of areas.

The following example is given as an illustration of its use. The several values of $\omega(d\delta M)/dt$ being taken from the present calculation :—

$\omega \frac{d\delta M}{dt}$.	
″	
−24·00	−24·00
−27·29	109·16
−30·72	61·44
−34·18	136·72
−37·62	75·24
−40·92	163·68
−43·91	87·82
−46·64	186·56
−48·99	97·98
−50·87	203·48
−52·35	104·70
−52·99	211·96
−53·13	−53·13
	−1515·87
	$\delta M = -505·29$

On application of the usual formula, involving finite differences, we get $\delta M = -505·31$; which represents the summation of the variations of M between the dates 1922 August 19 and 1924 September 25.

The Melbourne Observatory,
South Yarra, Victoria:
1923 *February* 20.

R Vulpeculæ. *Ch.* 7560. *H.* 205923. *A short analysis of the observations of Knott and others.* By Edward Prentice.

Knott's first observation of this star as given in *R.A.S. Mem.*, **52**, was on 1861 September 7, and with the exception of the years 1874, 1875, he carried on these observations annually till the end of 1892. The star's period being a comparatively short one, we thus have a good many periods available for analysis, especially when we bring into the field the observations by the members of the B.A.A., which observations commenced in 1911. In making a harmonic analysis of the light curves for the purpose of comparing Knott's results with those of subsequent observers, a difficulty presents itself in the magnitudes of the comparison stars used by Knott being slightly different from the Harvard values, and as I had doubts about the identity of all the comparison stars, his observations have been taken as they appear in the *Memoirs*. Light curves, too, drawn from a single observer, will probably differ somewhat from the mean of several observers, especially as regards the range.

There are several gaps in Knott's observations, but by taking groups of periods where sufficient observations are available, we arrive probably at fairly representative curves.

The results of the harmonic analysis are put in the form

$$c_1 \cos(\theta - \phi_1) + c_2 \cos(2\theta - \phi_2) + c_3 \cos(3\theta - \phi_3),$$

and taking for Knott's observations the zero starting-point at J.D. 2401003, the phases of the first harmonic are in good accord with a period of 136·6 days. These appear in Table I., together with the values of the coefficient of $\sin\theta$ in the expression $A \sin\theta + B \cos\theta$, showing an oscillation in the lengths of the periods spread over a considerable time.

For the B.A.A. observations, if we make J.D. 2419201 the zero starting-point, and adopt a period of 136·7 days, we get the phases agreeing pretty nearly with the period (Table I.). The starting-point here differs by three days from the epoch of maximum (E. 122) J.D. . . . 9198 as calculated from the curve. This group of phases gives evidence of an oscillation, but of a much shorter length than that shown by Knott's figures.

TABLE I.

Knott.				*B.A.A.*		
E.		ϕ_1.	Coeff. $\sin\theta$.	E.	ϕ_1.	Coeff. $\sin\theta$.
		°			°	
−11 to − 3		174	+ ·30	122 to 127	171	+ ·45
− 3 to	5	203	−1·15	127 to 131	184	− ·20
5 to	13	183	− ·15	131 to 135	188	− ·40
13 to	20	178	+ ·09	135 to 139	180	− 0
30 to	36	169	+ ·58	139 to 144	176	+ ·21
37 to	45	164	+ ·80			
45 to	53	166	+ ·67			
53 to	60	177	+ ·15			
61 to	68	182	− ·09			
68 to	73	204	−1·16			
Mean		180	+ ·004		179·8	+ ·012

N.B.—The coefficients A, B, C, . . . have been multiplied by 6 and divided by the range.

From his investigation of this star Professor Turner found the evidence consistent with an increase of period (*M.N.*, **80**, 493), so that the figures arrived at in this analysis confirm his deduction, though the increase in period is very small.

Since the date when Knott began his observations a series of oscillations have evidently taken place in the length of the period, but the oscillations have varied in extent, which tends to obscure any secular increase when a comparatively few periods only are being dealt with.

Phillips, in his Presidential Address to the British Astronomical Association, gave diagrams of the general trend of the oscillations of maximum in several stars (*B.A.A. Journ.*, **27**), and of those stars T Herculis, with a period of 165 days, seems to resemble this one in the fluctuations of the length of the period. The results of the analysis commencing with Knott's observations, after which there is unfortunately a considerable gap, is given in Table II., the first harmonic being in the form $C_1 \cos(\theta - 180°)$. A kind of serial run is noticeable in Knott's phases, marked *a*, *b*, *c*, *d*.

TABLE II.

E.	ϕ_2.	ϕ_3.		c_1.	c_2.	c_3.	Max.	Min.
Knott.								
− 11 to − 3	38°	294°	(*a*)	2·98	·10	·18	8·0	13·1
− 3 to 5	5	194	(*b*)	2·94	·46	·17	7·7	12·9
5 to 13	2	221	(*c*)	2·94	·20	·06	7·8	12·9
13 to 20	352	69	(*d*)	3·06	·55	·08	8·1	13·2
30 to 36	31	270	(*a*)	3·06	·38	·12	7·9	12·7
37 to 45	14	203	(*b*)	2·87	·50	·21	8·0	12·8
45 to 53	355	185	(*c*)	2·81	·40	·17	7·9	13·1
53 to 60	10	120	(*d*)	3·02	·33	·09	8·0	12·9
61 to 68	23	284	(*a*)	3·05	·42	·12	8·0	12·7
68 to 73	55	202	(*b*)	2·83	·51	·20	8·0	12·7
Harvard.								
97 to 105	40	153		2·85	·21	·16	8·0	12·5
108 to 122	48	286		2·95	·34	·06	8·1	12·8
B.A.A.								
122 to 129	19	163		2·93	·32	·09	8·1	12·4
129 to 137	17	75		3·06	·03	·04	8·0	12·4
137 to 145	27	200		2·98	·27	·14	7·9	12·3

In the last two columns the mean maximum and minimum magnitudes, as read from the ordinates of the curves, are given; these rather indicate an increase of brightness during the minimum phase, which may be due partly to differences in the values of the comparison stars made use of, and partly to the curves differing as previously mentioned.

The values of the fainter comparison stars made use of by Knott and the Harvard equivalents used by the B.A.A., if the identity is correct, are :—

Knott.	B.A.A.
$k = 10{\cdot}9$	11·19
$l = 11{\cdot}4$	11·37
$m = 11{\cdot}9$	11·7
$n = 12{\cdot}6$	12·5

The observed dates of maxima arrived at for the B.A.A. series of observations are :—

Epoch.	J.D.	Epoch.	J.D.
122	2419198	134	0845
123	9334	135	0984
125	9614	136	1117
126	9748	137	1252
127	9880	138	1394
128	20020	139	1522
129	0156	140	1656
130	0299	141	1794
131	0440	142	1936
132	0567	143	2071
133	0700	144	2206

Ashtead :
1923 *April* 2.

Note on the Occultation of Mars, 1923 *January* 22.

The occultation of Mars was observed at the Sydney Observatory on 1923 January 22.

Instrument—11¼-inch equatorial.

Immersion—Sky entirely overcast.

Emersion—Moon visible through a break in the clouds. Quite clear and good observation.

First seen January 21, $21^h\ 00^m\ 01^s{\cdot}1$ G.M.T.

External contact „ $21^h\ 00^m\ 19^s{\cdot}6$ „

There is reason to believe that the adopted longitude of the Sydney Observatory, viz. $10^h\ 04^m\ 49^s{\cdot}54$ is about half a second too far east, and that $10^h\ 04^m\ 49^s{\cdot}0$ is more accurate. If we use this latter value the times are

$21^h\ 00^m\ 01^s{\cdot}6$ G.M.T.

$21^h\ 00^m\ 20^s{\cdot}1$ „